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The sun eclipse of June 16th, 364 AD, as calculated by Theon of Alexandria in the 6th book of his Commentary on the Almagest.

Date : 24 Thoth 1112 Nabonassar, vague year
22 Payni 1111 Nabonassar „alexandrian" year
22 Payni 1111 Diocletian id. („era of martyrs)

Observed time, according to Theon : 1st contact : $2\frac{1}{2}\frac{1}{3}$ hours PM (<u>seasonal</u> hours) (ἀσφαλέστατα)
middle : $3\frac{1}{2}\frac{1}{4}\frac{1}{20}$ PM "
last contact : $4\frac{1}{2}$ PM (ἔγγιστα)

True conjunction according to the Almagest : (cfr Almagest ed Heiberg, pt I p. 466)

	Days of Thoth	Sun from apogee	Moon anomaly	Moon latitude
1101 years :	22; 41, 45	19; 11, 56	222; 53, 32	65; 41, 57
11 years	1; 9, 39	358; 28, 11	271; 4, 19	211; 12, 3
	23; 51, 24	17; 40, 7	133; 57, 51	276; 54, 0
	2×51,24 =	+65; 30 (apogee)	distance from sun:	−3; 50, 8
mean moon:	102,48	83; 10, 7	3; 8, 48	273; 3, 52
23; 10,7	×1/5 = equin. hrs	mean sun: Gemini	×1/12 = 0;15,44	+ 3; 24, 32
correction:	$20\frac{1}{2}\frac{1}{15}$	23; 10, 7	tot. 3; 24, 32	276; 28, 24
−3; 50 8	$+ 5\frac{1}{2}\frac{1}{3}\frac{1}{30}$	correction:	moon's speed:	
true position:	total: days 24	0;41,20	0; 34, 56	
Gem 19;19,59	Eq. hours $2\frac{1}{3}\frac{1}{10}$	true position: Gemini	time between mean and	
+ 3; 24, 32	$-0^h\frac{1}{3}\frac{1}{15}$	22; 28, 47	true conjunction:	
long. of moon at		+ 0; 15, 44	$5^h\frac{1}{2}\frac{1}{3}\frac{1}{30}$	
time of true conjunction:	total: equinoctial	true longitude at time	133; 57, 51	
Gemini	hours:	of true conjunction:	+ 3; 11, 36	
22; 44, 31	$2^h\frac{1}{2}\frac{1}{3}$	Gem. 22; 44, 31	137; 9, 27	

Results : Date of true conjunction : 24 Thoth $2\frac{1}{2}\frac{1}{3}$ equinoctial hours after midday of 24 Thoth

Positions : sun's and moon's centers : Gemini 22° 44' 52"
moon's anomaly 137; 9, 27 from apogee of epicycle
moon's latitude : 276; 28, 45 from northern extremity of orbit
moon's speed : 0° 34' 56" per hour = V

Apparent conjunction.

1° Vertical parallax of moon and sun at time of true conjunction :
(cfr Alm. I p. 174. Table of zenithal distances, „climate" of Lower Egypt)
Gemini 22; 45 hour : $2\frac{1}{2}\frac{1}{3}$ PM
angle of vertical and ecliptic at the point occupied by the moon (neglecting latitude) $\alpha = 17°35'$
zenithal distance of moon (neglecting latitude) : 38° 28'
(cfr Alm. I p. 442. Table of parallaxes)
for zenithal distance 38° 28' sun's vertical parallax : 0° 1' 45"
moon's vertical parallax : 0° 39' 35"
Difference : D = 0° 37' 50"

2° Component in longitude of the Difference : p = 0° 36' 1"

formula : $\frac{120}{\text{crd}(180 - 2\alpha)} = \frac{D}{p}$ (resolution of rectilinear rectangle triangle)

3° Time of apparent conjunction, 1st approximation. $\frac{p}{V} = \frac{0°36'1"}{0°34'56"} = 1\frac{1}{30}$ hour

$2\frac{1}{2}\frac{1}{3} + 1\frac{1}{30} = 3\frac{1}{2}\frac{1}{3}\frac{1}{30}$ PM

4° Vertical parallax of moon. (2nd approximation). (cfr Alm I p. 174 and 442)

time: $3\frac{1}{2}\frac{1}{3}\frac{1}{20}$ PM position: Gemini 22°45' (moon's movement will be accounted for later on).

angle of vertical and ecliptic: $\alpha =$ 18° 32'

zenithal distance: 51° 48'

vertical parallax of sun 0° 2' 15"

of moon 0° 49' 47"

difference: 0° 47' 32"

5° Component in longitude of difference: c = 0°45'3"

6° Epiparallax (empirical correction to dispense with recalculating parallaxes for a 2nd or 3rd approximation of time) = e (p: cfr 2°, supra)

p − c = d = 0°9'2" $\frac{p}{d}=\frac{d}{x}$ x + d = e = 0°11'17"

7° $\frac{13}{12}(p+e)$ = 0°51'18" (including empirical correction to account for moon and sun's movement)

8° Time of apparent conjunction 2nd approximation.

$\frac{0°51'18"}{0°34'56}$ = $1\frac{1}{2}$ hour, to add to hour of true conjunction.

$2\frac{1}{2}\frac{1}{3} + 1\frac{1}{2} = 4\frac{1}{3}$ PM

corresponding Position of moon: longitude: Gemini 22°44'55" + 0°51'18" = 23°36'10"

latitude: 276° 28'45" + 0°51'18" = 277° 20'3"

anomaly: 137° 9' 27" + 0°51'18" = 138° 0'45".

Circumstances of the eclipse.

(cfr Tables of eclipses, Alm I p. 519)

1° Argument: apparent „position of moon in latitude" at time of apparent conjunction

moon's vertical parallax at time of apparent conjunction: 0° 53' 17"

sun's — — — — — — 0° 2' 25"

Difference: 0° 50' 51"

Component in latitude of difference: 0°17'17"

which corresponds, on the oblique orbit of the moon, to a switching of: 0°17'17" × $11\frac{1}{2}$ = 3°19'

(in the Almagest I 530, 25, Ptolemy takes a coefficient 12).

„Position of moon in latitude", to be used as argument: 277° 20' + 3°19' = 274° 1'

2° Table for moon's maximum distance: argument 274°1' digits: 3; 58

path of moon: 0°23'21"

3° Table for moon's minimum distance: digits: 4; 46

path of moon: 0°25'56"

differences: digits 0;48, path: 0°2'35'

4° Table of correction for moon's intermediate positions:

Argument: moon's anomaly at time of apparent conjunction: 138° 0'45".

corresponding coefficient of correction: 0°51' 38"

0;48 × 0;51,38 = 0;41,18 correction for digits

0;2,35 × 0;51,38 = 0;2,13 correction for path

5° Results: 1st approximation of digits: 0;41,18 + 3;58 = 4;39,18

path: 0;2,13 + 0;23,21 = 0;25,34

empirical correction to account for the sun's move: 0;25,34 × $\frac{13}{12}$ = 0°27'42" moon's speed: 0°34'46" (as before). [The speed is supposed to remain the same during the whole time]

half duration of eclipse: $\frac{27'42''}{34'46''} = \frac{1}{2}\,\frac{1}{4}\,\frac{1}{20}$ hour.

First approximation of time table:

	equinoctial hours	longitude	anomaly
		Gemini:	
1st contact	$3\frac{1}{2}\,\frac{1}{20}$ PM	23°8'28"	137°33'3"
maximum phase	$4\frac{1}{3}$ PM	23°36'10"	138°0'45"
last contact	$5\frac{1}{10}\,\frac{1}{30}$ PM	24°3'52"	138°28'27"

Second approximation, taking into account the variation of zenithal distance during the eclipse. Theon uses here the method of the Handy Tables, but uses the tables of Almagest.

1° At approximate time of 1st contact, component in longitude of difference between sun and moon's parallaxes: (calculated as above) 42'13"

item at time of conjunction: 47'52"

difference d' = 5'39"

the moon's path, in 1st approximation, was 25'34"

empirical correction for further variations of zenithal distance: $\frac{25'34''}{5'39''} = \frac{5'39''}{x}$ $x = 1'$

25'34" + 5'39" + 1' = 32'13"

[this corrects the explanation I gave in 1931 in my edition of Pappus, p LXXIX: instead of $c + d' + x' + a$ it should be $d' + x' + a$]

Correction for the movement of sun: $\frac{13}{12} \times 32'13'' = 34'54''$ = new approximation of moon's path.

moon's speed (still supposed to have remained unchanged) 34'56" per hour

Duration of immersion, 2nd approximation: $\frac{34'54''}{34'56''} = 1$ hour.

2° The same calculations are repeated with the approximate time of last contact.

Result: $\frac{1}{2}\,\frac{1}{3}\,\frac{1}{12}$ hour = duration of emersion 2nd approximation.

(Rule of thumb: The longer time is the time of phase closer to midday)

Time table, new approximation.

	equinoctial hours	seasonal hours
1st contact	$3\frac{1}{3}$ PM	$2\frac{1}{2}\,\frac{1}{3}$ PM or $8\frac{1}{2}\,\frac{1}{3}$ from sunrise.
maximum phase	$4\frac{1}{3}$ PM	$3\frac{1}{2}\,\frac{1}{4}\,\frac{1}{20}$ PM or $9\frac{1}{2}\,\frac{1}{4}\,\frac{1}{20}$ from sunrise.
last contact	$5\frac{1}{4}$ PM	$4\frac{1}{2}$ PM or $10\frac{1}{2}$ from sunrise.

Prosneusis.

[in the actual state of my edition, the figures of the manuscripts do not fit together, nor with the preceding ones. Maybe they will be all right when the work is terminated. In the mean time, I continue the calculations, following the methods explained by Theon, but putting figures taken from the Almagest ed Heiberg, or the Handy Tables ed Halma]

I make the calculation for 1st contact only, from the following data:

Hour of 1st contact: $2\frac{1}{2}\,\frac{1}{3} = 2^h\,50^m$ seasonal time PM

sun's longitude, at time of conjunction (true conjunction) Gemini 22°44' or 45'

moon's „latitude": 276°28'45" from northern extremity. Consequently the moon is north of the ecliptic, having covered more than 3/4 of its orbit.

1° Length of seasonal hour. cf. Alm. I p. 134. „Climate" of Lower Egypt.

AR Gemini 20 ~ 79 5
30 ~ 90 0
22°44' ~ 82°3' by interpolation.
Oblique ascension Gem 22°44' ~ 67° 17' idem

82°3' − 67°17" = 14°46'
14°46' : 6 = 2°27'40"
15° + 2°27'40" = 17°27'40" or 17°28'

2° Ecliptic's rising point: time $2^h 50^m$ seasonal
2;50 × 17;27 = 49°26'30"
Sun's oblique ascension 67°17' (as above Gemini 22°44')
obl. asc. of rising point: 116°44'
Oblique asc. Lion 10 121 32'
0 109 37
10 11 55
2 7 7

$\frac{10 \times 7;7}{11;55} = \frac{1,11;10}{11;55} = 5;58$

Oblique asc. Lion 5°58' = 116°44'

(cf Almagest ed Heiberg vol I plate at end of volume)
rising point : Lion 5°58' azimuth of d° 21°50' N from true E
setting point : Aquar. 5°58' azimuth of d° 21°50' S from true W.

3° Table of prosneusis cf Alm I 544
maximum of eclipse at Alexandria: 4;39 digits.
4 d ~ 42°56'
5 d 36°35'
4;39 38°4' northern direction from W = „prosneusis"
azimuth of rising point: −21°50' southern direction from W
16°51' northern direction from W = prosneusis at 1st contact
[The calculation of last contact's prosneusis is done along the same way]

Sample of sexagesimal division:

1	11	55
2	23	50
3	35	45
4	47	40
5	59	35
6	1 11	30
7	1 23	25
8	1 35	20
9	1 47	15
10	1 59	10
20	3 58	20
30	5 57	30
40	7 56	40
50	9 55	50
60	11 55	0

1 11 10 | 11 55
11 35 0 | 5; 58
1 39 10
3 50

method of Theon (or Hypatia) 3rd book, to be used with an abacus, which gives mechanically the table and the result.

The sun eclipse of June 16 364, as calculated by Theon of Alexandria according to the Handy Tables, in the 6th Book of his Commentary on the Almagest.

Date : 688 (vague year) of the era of Philippus. (cfr ed Halma pt 2 p.67)

	Sun from apogee	Excentric's apogee	Epicycle's center	moon's center	"latitude" from N extremity
676 (years)	358 4	262 3	291 13	210 26	320 30
12 (years)	357 5	118 23	230 57	344 37	232 0
Thoth	0	0	0	0	0
23d (day)	21 41	246 31	176 24	287 26	1 10
21 (hour)	0 52	9 48	21 20	11 26	0 3
1/10 (hour)	0 0	0 <2>	0 6	0 3	0 0
	17 41	276 47	360 0	133 58	193 43

correction 41'
tempor. 17°0'
+ 65°30'
82°30'
Gemini 22°30'

mean position
Sun Gemini: 23 11 − 0 42 = 22°30' true
moon: 23 11 − 3 50 = 19 21'

true distance at mean conjunction: 3°8'
3°8' × 1/12 = 0°15'40"
3°8' + 0°15'40" = 3°23'40"
true conjunction : Gemini 22°44'40"

Alternative way, according to the Small Commentary on the Handy Tables by Theon.
(Interesting because it is the system of epacts of the church calender.) cfr ed Halma pt 1, p 69.

Fixed year, Diocletian 80 month Payni. or 687 Philippus fixed year.

687 : 19 rest : 3 or (80−1) : 19 rest 3 (the "golden number of the church calendar)

3 × 11 = 33 = epact (Theon calls it epact Eo)

Supplement for Payni (10th month) 10 : 2 = 5

5 + 3 = 8 30 − 8 = 22

22
+ 97 (number of "bisextile" years since the introduction of the system)
119

Counting 119 days from Payni we come to Thoth 24 of the (next) (vague) year 688 Philippus

This enables us to discriminate the result of the above calculation which might as well have given the date of a full moon. If we don't use the epact, there is another way of discriminating.

[I cannot yet guarantee that this alternative way is not a later interpolation, but it seems prima facie, to be authentic

True conjunction.

mean sun at time of mean conjunction :	Gemini 23°11'	
correction	0°42'	
true sun at time of mean conjunction	Gemini 22°19'	as found above
true moon at time of mean conjunction	Gemini 19°21'	as found above
Distance sun − moon :	3°8'	

Empirical correction to account for sun's move : $3°8' \times \frac{13}{12} = 3°24'$

Moon's speed (cfr προχαρόνιον ed Halma pt 1 p 147) (empirical determination)

position of moon's center on the epicycle 133°58'

corresponding number in προχαρόνιον 51 $\frac{51}{10} = 5$ $V = 30 + 5 = 35'$ speed per hour

Time between mean and true conjunction: $\frac{3°\,24'}{0°\,35'} = 5\,\frac{1}{2}\,\frac{1}{3}\,\frac{1}{30}$ hours.

Time of mean conjunction, as above: 23 Thoth $20\,\frac{1}{10}$ hours PM. mean time
or $20\,\frac{1}{2}\,\frac{1}{3}\,\frac{1}{10}\,\frac{1}{60}$ PM true time, equinoctial hours.

Note: The notion of equation of time used here is somewhat different from ours.

Time of true conjunction: $20\frac{1}{2}\frac{1}{3}\frac{1}{10}\frac{1}{60} + 5\frac{1}{2}\frac{1}{3}\frac{1}{30} = 2\frac{1}{2}\frac{1}{3}$ hours after midday of 24 Thoth.

Corresponding positions: sun and moon's longitude: Gemini 22° 44'
moon's „latitude" from northern extremity 276° 28'
moon's center on the epicycle 137° 9'
neglecting a correction for the moon's excenter, as is always done, according to Pappus and Theon.

Apparent conjunction.

1° Difference of sun and moon's parallaxes, reduced in latitude and longitude,
given directly by the handy table of parallaxes, cfr Halma pt 2 p. 99 and pt 1 p 147.
$p = 35'$ component in longitude.

2° Epiparallax. (moon's speed $V = 35'$ per hour as above)
1st approximation of apparent conjunction: $\frac{p}{V} = \frac{35}{35} = 1$ hour
$2\frac{1}{2}\frac{1}{3} + 1 = 3\frac{1}{2}\frac{1}{3}$ PM
component in longitude of difference between sun and moon's parallax, for $3\frac{1}{2}\frac{1}{3}$ PM: $c = 38'$
$p - c = d = 8'$
$\frac{p}{d} = \frac{d}{x}$ $\quad x + d = e = 10'$ $\quad e + p = 45' =$ epiparallax
empirical correction for sun's move: $\frac{13}{12} \times 45' = 49'$

3° Elements of eclipse:
Time of apparent conjunction $\frac{49'}{35'} = 1\frac{1}{3}\frac{1}{15}$ hour (= time between true and apparent conjunction)
(moon's speed) ⟶
$2\frac{1}{2}\frac{1}{3} + 1\frac{1}{3}\frac{1}{15} = 4\frac{1}{4}$ PM, time of apparent conjunction.
Corresponding positions of moon: longitude Gemini 23° 34'
position in „latitude" from northern extremity: 277° 17'
Consequently: moon's true latitude: 0° 38' N
Component in latitude of difference between moon and sun's parallaxes, given by handy table: 17'30"
apparent latitude of moon: 0° 38' − 0° 17' 30" = 0° 20' 30" N

4° Circumstances of the eclipse (cfr Halma II p. 91 and I p 147 = προχειρον)
digits: 4; 58
moon's path: 26' 39"
$\dfrac{\frac{13}{12} \times 26'\,39}{35'\ \text{(moon's speed)}} = \frac{1}{2}\frac{1}{3}$ hour = duration of immersion and emersion, 1st approximation.

5° Time table, 1st approximation.

	Hours	Positions
1st contact	$3\frac{1}{3}\frac{1}{12}$ PM equinoctial hours	Gemini 23° 5' 8"
maximum phase:	$4\frac{1}{4}$ PM	23° 34'
last contact:	$5\frac{1}{12}$ PM	24° 2' 52"

Reduction of digits in „square digits" ($\frac{1}{12}$ of sun's surface) $3\frac{1}{2}\frac{1}{7}$ sq.d.

Second approximation

a/ Component in longitude of difference between sun and moon's parallaxes at approximate times of:

1st contact : $c' = 0°35'$

apparent conjunction : $c = 40'$

$c - c' = 5' = d'$ moon's path $= a = 26'39''$ as above.

$\frac{a}{d'} = \frac{d'}{x}$ $x = 1''$ which is neglected.

$a + d' + x = 31'39''$ $\frac{13}{12} \times 31'39'' = 34'17''$ $\frac{34'17''}{35' \text{ (moon's speed)}} = 1$ hour

b) component in longitude of difference between sun and moon's parallax at approximate time of:

last contact : $c'' = 43'$

$c - c'' = d'' = 3'$ (absolute value)

$\frac{a}{d''} = \frac{d''}{x}$ $x = 0'20''$ neglected.

$a + d'' + x = 30'$ $\frac{13}{12} \times 30 = 32'30''$ $\frac{32'30''}{35'} = \frac{1}{2}\ \frac{1}{3}\ \frac{1}{15}$ (last figure suspect)

Time table.

	equinoctial hours	seasonal hours	idem from sun rise
1st contact:	$3\frac{1}{4}$ h PM	$2\frac{1}{2}\frac{1}{3}$ PM	$8\ \frac{1}{2}\frac{1}{3}$
maximum phase	$4\frac{1}{4}$	$3\frac{1}{2}\ \frac{1}{6}$	$9\ \frac{1}{2}\frac{1}{6}$
last contact	$5\frac{1}{6}$	$4\frac{1}{2}$	$10\ \frac{1}{2}$

Prosneusis

Theon gives no figures in his commentary on the Almagest. I make here the calculation according to his explanations, with figures from the handy tables.

Data: 1st contact $2^h 50^m$ seasonal time PM. climate of Lower Egypt.

Sun's longitude at time of true conjunction : Gemini $22°44'$ or $45'$.

1° Length of seasonal hour: given directly by the Handy Table of oblique ascensions: $17°28'$.

$17°28' \times 2;50 = 49;29,20$.

2° Sun's oblique ascension (Gemini $22°45'$) $\sim 67°16'$

$49°29' + 67°16 = 116°45'$ oblique ascension of rising point, which is consequently: Lion $5°56'$

Setting point : Aquarius $5°56'$

3° azimuth of setting point : $a = 21°51'$ South (taken in the Almagest)

4° " arc e " given by handy table of parallaxes column 4. climate of Lower Egypt, Gemini $22°45'$

time : $2^h 50^m$ seasonal, $= 3^h 38^m$ PM equinoctial

$e = 75°3'$.

5° arc b given by handy table of prosneusis

for an eclipse of $4;58$ digits: $b = 37;12$.

6° The latitude of moon being north, the formula to be used is:

$$\frac{b \times (180 - e)}{90} = \frac{37°12'\ (180 - 75°3')}{90°} = 43°32'$$

$43°32' - 21°51' = 21°41'$ prosneusis, to be counted from true W in direction of N.